THE ROLE OF PITCH AND ROLL EFFECTS ON THE SHORT PERIOD INSTANTANEOUS SEA LEVEL HEIGHT DETERMINATION WITH GPS

Dr. Reha Metin ALKAN

Department of Geodesy and Photogrammetry Eng.,
Istanbul Technical University, Turkey.

Abstract

Depth measurements are carried out from the Instantaneous Sea Level (*ISL*) during the survey intervals. Water in oceans, seas and lakes move due to the variety of reasons, thus it is time and space-dependent. It is obvious that, these depth values should not be used with a different water level if they are not tied to any stationary (vertical) datum. For this purpose, a tool called tide gauge is commonly used. On the other hand, the *ISL* could be determined by vessel-borne GPS method. In this method, a vessel which is equipped with the GPS receiver is used as a gauge. However, the surveying boat or vessel moves irregularly by the effects of waves or other disturbing factors even when it is anchored. The goal of this study is to introduce a method which determines and removes the errors resulted from the *P&R* effects with low-cost inertial measurement device (two axis tilt sensor). At the end of the test measurements, the *P&R* errors are removed from the GPS-derived ellipsoidal heights via introduced method in the paper. The results show that the effect of the *P&R* errors on the mean value of the h_{ISL} is reached 2 and 5 centimeter for the first and second test measurements, respectively, even measurement environment is relatively calm. When considering the use of vessel-borne GPS in offshore hydrographic surveying, it could be clearly said that, these effects will influence the measurements more significantly.

1. Introduction

Depth measurement is realized from the Instantaneous Sea Level (*ISL*) during the surveying interval. Water in oceans, seas and lakes move due to the variety of reasons, thus it is time-dependent. Changes vary up to several meters and depend on the region and water area. They can be seasonal, monthly even daily according to the region. Meteorological (i.e. pressure, winds) and oceanographic effects, exceptional vertical crust movements and astronomical tides are the main reasons of these effects (Torge, 1991). Engineer needs to consider the local difference between the *ISL* and vertical datum when working on projects extending from land into the water (e.g. harbors, jetties, water and sewer outlets, etc.). For achieving that task and to establish a relation between the depths and any stationary datum, sea level should be monitored and recorded.

2. Review of the Sea Level Height Determination

Sea level changes should be monitored and recorded for reduction of the measured depths. For this purpose, a tool which is called gauge is commonly used. There are some kind of gauge types for the various aims and scopes; '*Staff Tide Gauge*' is commonly used in conventional hydrographic surveys with an accuracy of ±(1-5) cm approximately. '*Float Gauge*' is developed for more precise measurements in which the effects of wave movements are eliminated. Their accuracy is between ±(0.1-1) cm. On the other hand, ±(0.1-0.5) mm accuracy can be achieved by using '*Recordable Tide Gauges*' (Fig. 1). Modern tide gauges use less mechanical but more electronic components for measurement and in particular for recording the changes of water level in time. Besides of this, for monitoring the height changes of the global sea surface with an accuracy of ±(3-4) cm, the space-borne satellite altimeters such as TOPEX/Poseidon Follow-On satellites are used (Chamber *et al.*, 1999, Chen *et al.*, 2000). Also, precise sea level determination can also be realized by long-period differential GPS method (Parke *et al.*, 1997, Han *et al.*, 1999).

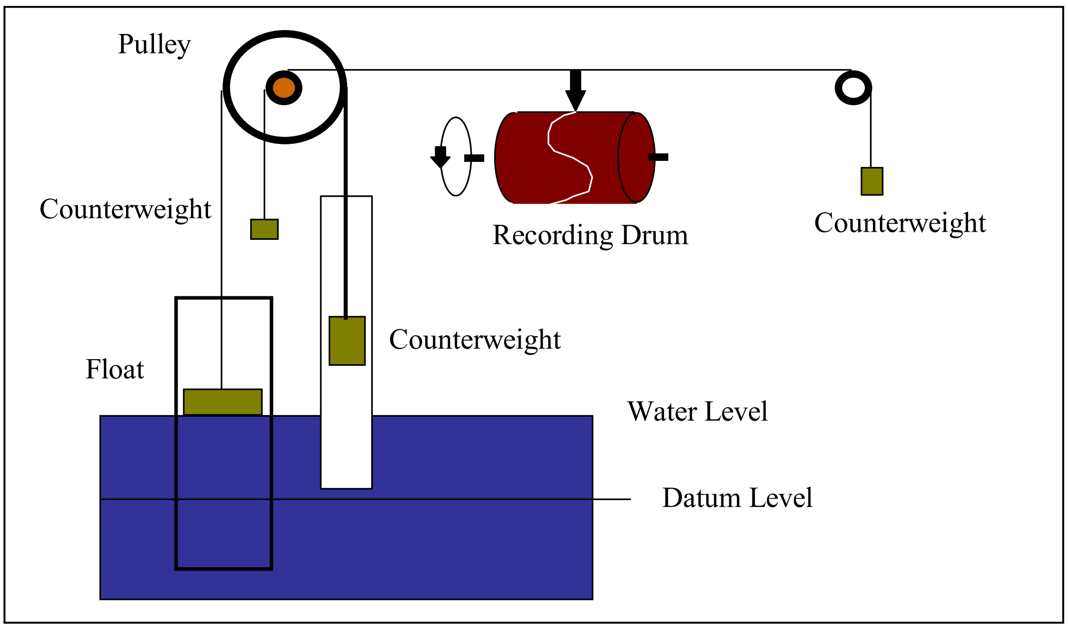

Fig. 1. Sea Level Changes Observation with a Tide Gauge

In most hydrographic applications, it is enough to use a staff tide gauge to determine the *ISL*. According to the new standards accepted by the International Hydrographic Organization (*IHO*) in 1997, the total measurement error of tidal heights should not exceed ± 5 cm at the 95% confidence level for '*special order*' surveys and ±10 cm for '*other*' surveys (Mills, 1998).

GPS-aided vessel can also be used as a tide gauge to determine the *ISL* (Alkan, 1998). The schematic depiction of this method is given in Figure 2.

The *ISL*'s orthometric height can be expressed by using ellipsoidal height obtained from the GPS geodetic coordinates and geoid undulation (from Fig. 2):

$$H_{ISL} = h_{ISL} - N \tag{1}$$

where, h_{ISL} is the distance of the *ISL* from the WGS-84 ellipsoid, i.e. ellipsoidal heights of the *ISL* and N is the relative geoid undulation of the area.

Fig. 2. A Schematic Depiction of Determination of the *ISL* with GPS-Aided Vessel

To obtain the ellipsoidal heights of the *ISL*, the vessel was anchored the place in the study area, and instantaneous ellipsoidal height for every measurement epoch ($h_i^{inst.}$) was measured. The h_{ISL} value was obtained by

$$h_{ISL} = \frac{\Sigma(h_i^{inst.})}{i}; \quad i = 1,2,\ldots,n \ (number\ of\ measurement\ epochs) \qquad (2)$$

Our experiences showed us that, about 30 to 60 minutes observation was enough to calculate the h_{ISL} value for short period surveying applications because there was no significant tide in application area. Data collection time could be varied depend on the geographical conditions and circumstances of the application area. By keeping the study area conditions in mind, this procedure could be repeated several times with certain intervals (for example every 4 or 6 hours) if necessary. In this way, the changes (rise and fall) that occur in the sea level during the day that are caused by tide or other effects will be considered.

Hydrographic survey vessel. shows irregular three-dimensional movements due to environmental effects, such as wind or tide. As a result of this, the vessel continues its movement with a heave motion in vertical plane and *Pitch and Roll (P&R)* angles in horizontal plane even when it is anchored. This motion causes some errors in depth and position. Magnitude of these errors might reach up to meters depending on the marine environment conditions. If the *ISL* height needed to be determined more accurately, the *P&R* effects should be considered and removed from the measurements.

3. Data Collection Methodology of The Fieldwork

The test measurements were performed in the Halic Bay, which is located in Bosporus of Istanbul, Turkey (Figure 3). Halic Bay, widely known as Golden Horn, is a natural bay, nearly 8-km in length, 20 to 700 m wide, and a maximum depth of 50-m. The measurements were performed in August 1999 and April 2000.

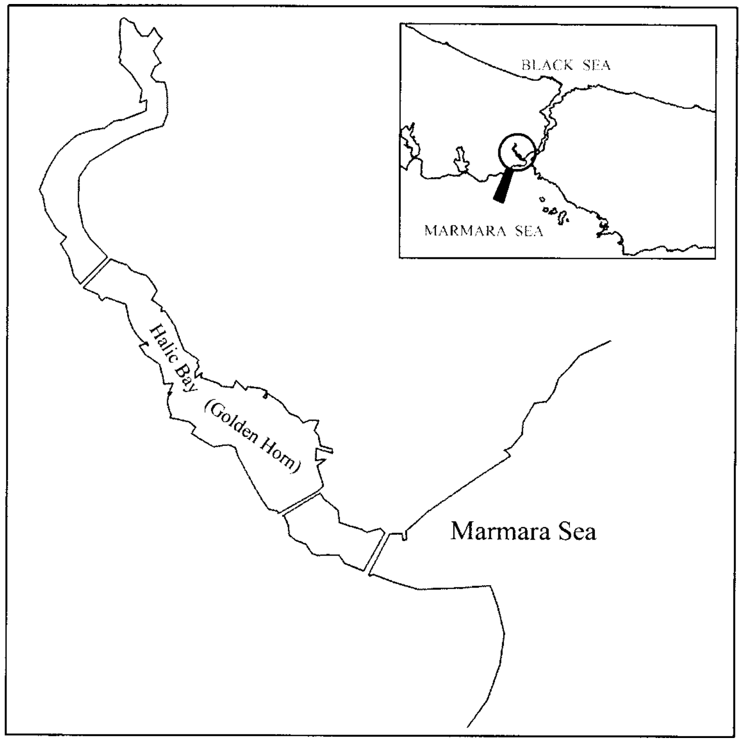

Fig. 3. The Fieldwork Area

The survey vessel was anchored on a place having calm water characteristics in the survey area and data-logging procedure was started. The raw data was collected at a 1 Hz rate with Leica System-300 receivers in kinematic mode. Geodetic latitude, longitude and ellipsoidal heights (in WGS-84 ellipsoid) were obtained. For the correction of the pitch and roll errors, they have to be measured. There are some kinds of devices which measure these angles together with heave motion in situ measurement on the vessel. They are commonly called digital heave compensator (or sensor). With the help of hydrographic software it is possible to import data directly from the sensor to the computer and correct the measured data for the heave, pitch and roll errors in real-time. The most important drawback of the devices is their high cost ($30000 USD~$50000 USD). Therefore, this hardware is not used widespread. In this study, the pitch and roll angles were measured by a linear dual axes tilt sensor at a 1 Hz rate. This low cost device has both analog and Rs232 outputs and measures the angles with an accuracy of 0.5% of the angle from -30 to +30 degrees intervals (AOSI, 2000). The angles are stored into a computer by means of a Visual Basic based program in order to be used in post processing stage.

4. Pitch and Roll Corrections

One of the major source of errors in hydrographic surveying arises from uncompensated vessel motion-heave, pitch and roll. Generally, a rigid object can be described by translation and rotation around the center of gravity of the body. Translation has three components: surge, sway and heave. Rotation has also three components, namely pitch, roll and yaw (Wiele, 2000 and Work *et al.*, 1998). Heave is described as the oscillatory rise and fall of a boat due to the entire hull being lifted by the force of the sea. Pitch is described as the oscillations of a ship around the transverse axis, due to the bow and stern being raised or lowered on passing through successive crests and troughs of waves, and roll is described as the oscillation of a ship around the longitudinal axis (IHO, 1994). The errors which depend on the motions can be significant, especially in wavy conditions. The *P&R* angles of the vessel along with heave motion must be measured in order to obtain accurate depth and position values.

To calculate the *P&R* corrections, a coordinate system has to be defined. The coordinate system used in the algorithm is shown in Figure 4. The center of gravity of the vessel (*COG*) is chosen as the origin (*O*) of the coordinate system and is determined under a technical collaboration with the Naval Architecture and Ocean Engineering Department of the Istanbul Technical University. The coordinate system was configured so that the heading of the vessel was the *x*-axis. The *x* and *y* axes of coordinate system were defined so that the *pitch angle* was positive when the bow of vessel was side down, and the *roll angle* was positive when port of vessel was side up.

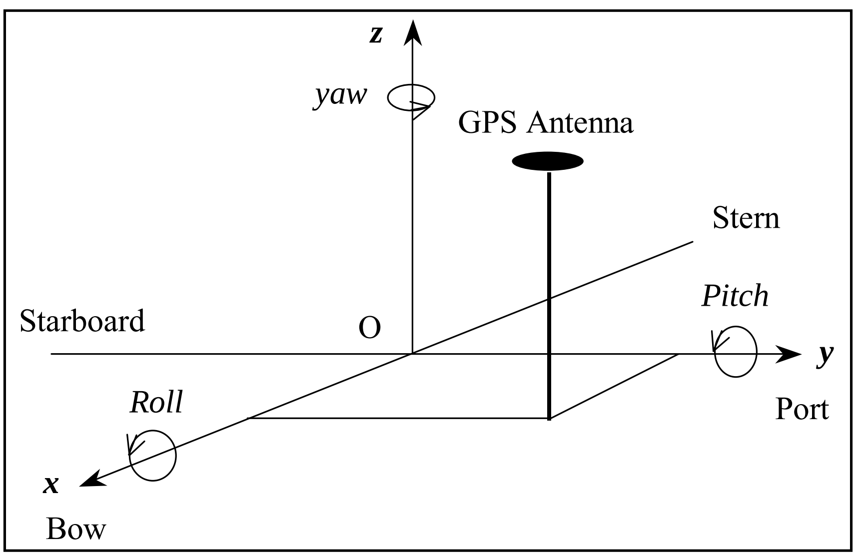

Fig. 4. Coordinate Frame

The ellipsoidal heights (and if necessary, other measurements) could be corrected for the *P&R* effects for every t_i measurement epoch by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{LL} = \underline{\Phi} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{BF} \quad (3)$$

where, $\underline{\Phi}$ is transformation (total rotation) matrix which is transform the body frame to the navigation frame, X_{LL}, Y_{LL} and Z_{LL} are the local level coordinate system and x_{BF}, y_{BF} and z_{BF} are the body reference (vessel-fixed) frame. The total rotation matrix can be expressed by (Anderson and Mikhail, 1998)

$$\underline{\Phi} = \begin{bmatrix} CosBCosP & SinBCosR + CosBSinPSinR & SinBSinR - CosBSinPCosR \\ -SinBCosP & CosBCosR - SinBSinPSinR & CosBSinR + SinBSinPCosR \\ SinP & -CosPSinR & CosPCosR \end{bmatrix} \quad (4)$$

where, B is the bearing angle of the heading (clockwise horizontal angle from the true north), P and R denote the *pitch* and *roll* angles obtained from the tilt sensor on the vessel, respectively. The *P&R* corrections for ellipsoidal heights which are used for the calculation of (h_{ISL}) are calculated by

$$Corr.^{h} = z_{BF} - (SinP\, x_{BF} - CosPSinR\, y_{BF} + CosPCosR\, z_{BF}) \quad (5)$$

The calculated corrections are plotted in Figure 5 and 6 for the first and second test measurements, respectively. Note that, in the first application, data were obtained with a rate of 1 second for approximately 30 minute, whereas in the second application for approximately 60 minute measurements.

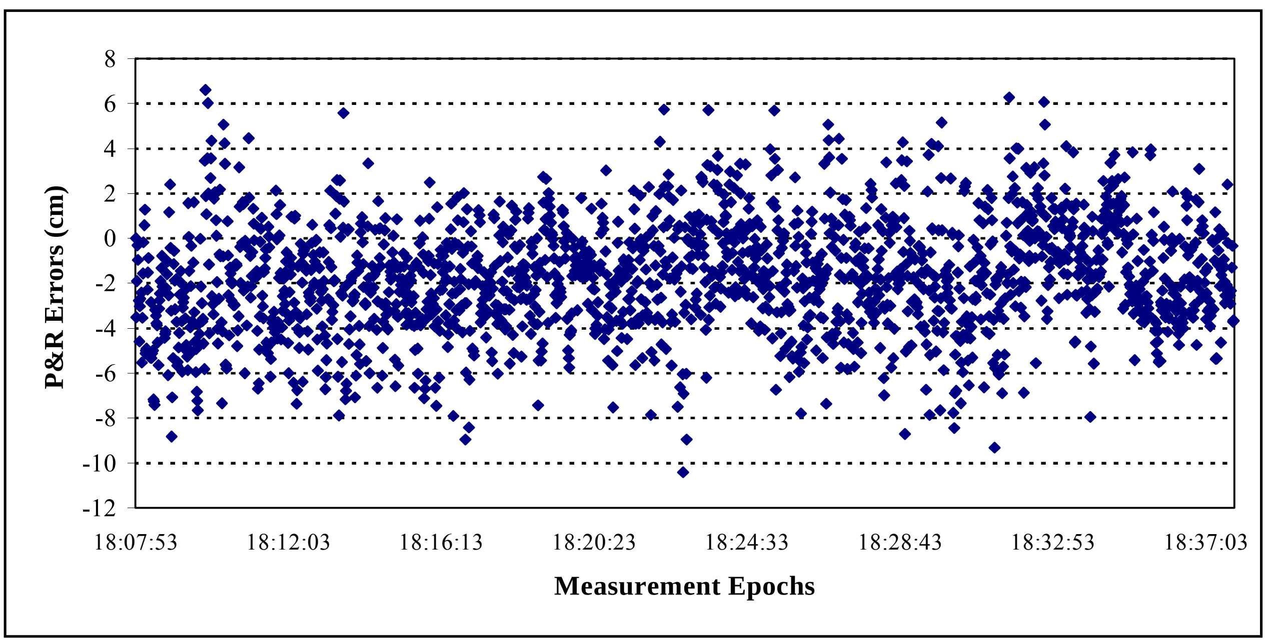

Fig. 5. The *P&R* Corrections for the Ellipsoidal Heights
Belong to the First Trial Measurement

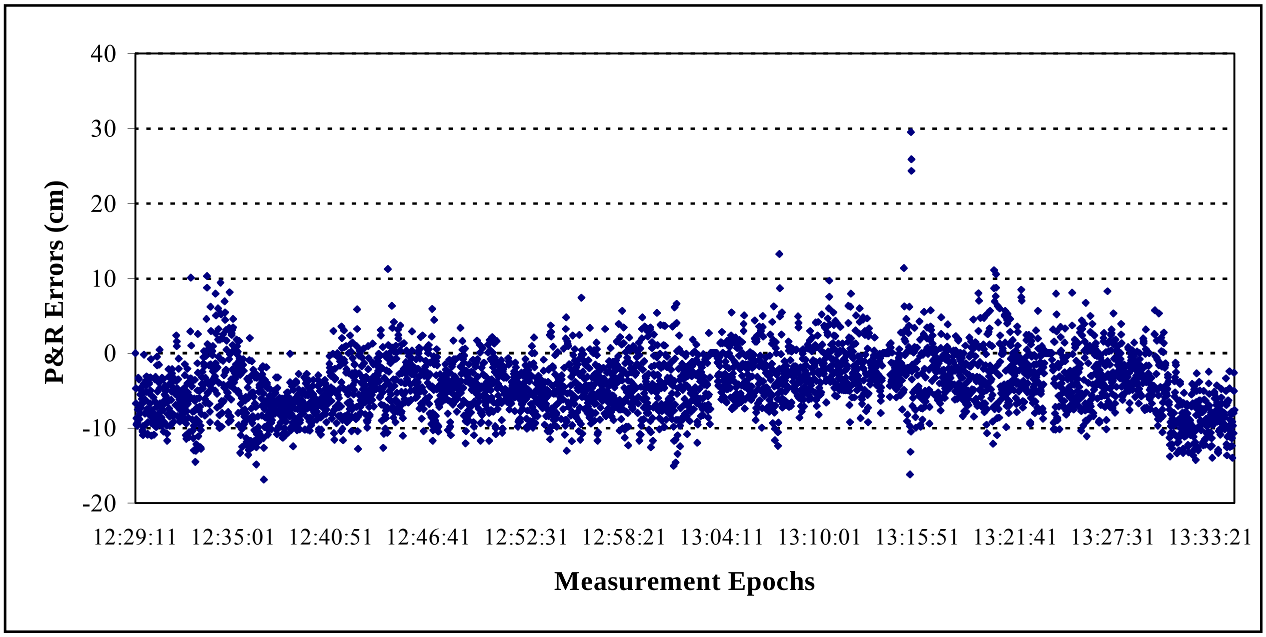

Fig. 6. The *P&R* Corrections for the Ellipsoidal Heights
Belong to the Second Trial Measurement

It is clearly seen from the Figure 5 and 6 that, there are significant correction values for pitch and roll effects. Indeed, calculated pitch and roll errors are in the intervals between -10 cm and +7 cm for the first application, and -17 cm and +30 cm for the second application. Value of the h_{ISL} is obtained from arithmetical average of the $h_i^{inst.}$ values according to the (2). The h_{ISL} is calculated from both uncorrected and corrected ellipsoidal heights for first and second applications, respectively. The corrected value of ellipsoidal heights are obtained from

$$Corrected\,(h_i^{inst.}) = h_i^{inst.} + Corr.^h \tag{6}$$

The results of the calculations are given in the Table 1.

Table 1. The Effect of the *P&R* Errors on the Mean Value of the h_{ISL}

	h_{ISL} from uncorrected $h_i^{inst.}$	h_{ISL} from corrected $h_i^{inst.}$	Differences
Firs Test	18.90 m	18.88 m	0.02 m

Measurement			
Second Test Measurement	18.78 m	18.73 m	0.05 m

5. Error Budget for Calculated Corrections

If error propagation low is applied to the (5) with respect to the independent variables, i.e. x_{BF}, y_{BF}, z_{BF}, P and R, the accuracy of the correction value can be estimated.

$$f(corr.^h) = f(x_{BF}, y_{BF}, z_{BF}, P, R) \tag{7}$$

$$\begin{aligned} df(corr.^h) = {} & \frac{\delta f(corr.^h)}{\delta x_{BF}} dx_{BF} + \frac{\delta f(corr.^h)}{\delta y_{BF}} dy_{BF} + \frac{\delta f(corr.^h)}{\delta z_{BF}} dz_{BF} + \\ & + \frac{\delta f(corr.^h)}{\delta P} dP + \frac{\delta f(corr.^h)}{\delta R} dR \end{aligned} \tag{8}$$

Then, magnitude of pitch and roll errors could be expressed by the standard deviation equation which is given below;

$$\begin{aligned} \sigma^2_{corr.} = {} & \left[\frac{\delta f(corr.^h)}{\delta x_{BF}}\right]^2 \sigma^2_{x_{BF}} + \left[\frac{\delta f(corr.^h)}{\delta y_{BF}}\right]^2 \sigma^2_{y_{BF}} + \left[\frac{\delta f(corr.^h)}{\delta z_{BF}}\right]^2 \sigma^2_{z_{BF}} + \\ & + \left[\frac{\delta f(corr.^h)}{\delta P}\right]^2 \sigma^2_P + \left[\frac{\delta f(corr.^h)}{\delta R}\right]^2 \sigma^2_R \end{aligned} \tag{9}$$

In order to calculate the magnitude of the error in the (9), standard deviation of the antenna coordinates ($\sigma_{x_{BF}}, \sigma_{y_{BF}}, \sigma_{z_{BF}}$) are estimated as 0.005 m and the standard deviation of the *P&R* angles (σ_P, σ_R) are estimated as 0.110 degree. At the end of the calculation, average of the standard deviations is found as 2 mm. Calculated value shows that; corrections are obtained with an order of high accuracy. It is important to state that, this value only represent accuracy of the corrections, namely not express accuracy of the ellipsoidal height measurements.

Statistical significance tests of the corrections for each ellipsoidal height were also performed. To determine the significance of each correction was tested using *t-distribution*. In this stage of the study, the null hypothesis $H_0 : E(Corr.^h) = 0$ was tested against the alternative hypothesis $H_0 : E(Corr.^h) \neq 0$. Test results showed that nearly all of the corrections were significant and should be applied to the relevant ellipsoidal heights.

6. Conclusions and Suggestions

GPS-derived ellipsoidal heights are successfully corrected for pitch and roll errors by a low-cost inclinometer (tilt sensor) with the algorithm presented in this paper. Results showed that the effect of the *P&R* errors on the mean value of the h_{ISL} was reached 2 and 5 centimeter for the first and second test measurements, respectively, even measurements were performed near the shore and in calm water environment. Corrections would reach more than one decimeter value if marine environment were wavy. When vessel-borne tide gauge method is used for offshore studies and surveying is performed in a strong wave environment, corrections could reach up to higher magnitudes. Briefly it can be said that, if *ISL* height is needed obtained with higher accuracy, the *P&R* errors had to be considered and their effects had to be removed from the measurements.

Appendix I: References

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Appendix II: Notation

The symbols used in this paper:

B	=	bearing angle,
$Corr.^h$	=	pitch and roll corrections for the ellipsoidal heights,
h_{ISL}	=	ellipsoidal height of the *ISL*,
$h_i^{inst.}$	=	instantaneous ellipsoidal height for every measurement epoch,
H_{ISL}	=	orthometric height of the *ISL*
N	=	geoid undulation,
P	=	pitch angle,
R	=	roll angle,
(x_{BF}, y_{BF}, z_{BF})	=	coordinates in the body reference frame,
(X_{LL}, Y_{LL}, Z_{LL})	=	coordinates in local level coordinate system,
σ	=	standard deviation,
$\underline{\Phi}$	=	rotation matrix.

The abbreviations used in this paper:

COG	=	Center Of Gravity,
ISL	=	Instantaneous Sea Level,
IHO	=	International Hydrographic Organization