Assessment of performance of ionospheric delay-free GPS carrier phase kinematic relative positioning with varying baseline lengths

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Abstract

It has been shown in the past that differential GPS carrier phase kinematic position accuracy tends to degrade as baseline length increases. This position solution degradation is due to several factors, primarily, errors attributed to the spatial variation of atmospheric delays (and secondly, errors in the satellite orbit). This paper endeavours to assess the magnitude of these effects on the position solutions using various base stations located at distances between 1 and 200+ km. away from the vessel. The assessment will be based on data collected on the Chesapeake Bay during July 1999, in three consecutive days. The data were collected by an Ashtech Z12 receiver mounted on the NOS S/V Bay Hydrographer. The baselines used in the assessment were processed using an ionospheric delay-free processing technique. A local truth trajectory was established while the vessel was close enough to base station TANG to employ integer fixed RTK. This procedure yielded results that agreed with the "truth" trajectory to the decimetre level in latitude, longitude and height.

Introduction

A variety of DGPS positioning applications take place in the offshore, sometimes at great distances from established reference stations. Often these applications require higher positional accuracies than achievable by code DGPS. The results presented in this paper will show that ionospheric delay-free processing techniques can provide decimetre level accuracies for baselines in the range of 47 to over 200 km.. More accurate results, sub decimetre at 95% confidence level, could be attained in some cases by employing a fixed integer technique. The ionospheric delay-free solution, however can provide decimetre level accuracies with none of the risks associated with integer fixing and over much longer baseline distances.

This paper provides a preliminary analysis of the data set. A portion of the data included in this examination was included in a paper by Huff and Remondi (2000) and could be explored further in future research. The expedition collected additional data not included in this analysis.

Tangier Island Expedition

The data set consists of three consecutive days of data collected on board a survey vessel, National Ocean Service (NOS) S/V Bay Hydrographer with a dual frequency Ashtech Z-12 receiver. Five stations, of known location in the Chesapeake Bay area, occupied by dual frequency receivers were used as reference stations (TANG, NASA, DENY, SSMC, and BWRX). TANG was used as the primary reference station and the others were determined relative to TANG. The relative height error for the other four base stations is not known but it is estimated to be smaller than 5 cm.. The data was collected from the 20th to the 22nd of July, 1999 and resulted in data set days 201, 202, and 203, each 18 to 24 hours in length. The data was collected at a rate of 1 Hz. on both the vessel and reference receivers.

Each day the data consists of several hours collected while the vessel was at the dock followed by a cruise. The duration, departure time, and trajectory of the cruise varied for each day. Only those epochs after solution convergence and up to where the vessel reaches a maximum of 12 km. away from the primary reference station TANG will be used in the analysis. This was done in order to disregard those measurements taken before a narrow lane fixed solution was achieved and those measurements taken beyond the accepted limits, in terms of baseline length, for reliable integer fixing. The result of this was a "truth" segment expected to be accurate at the centimetre level. As an example, the vessel's trajectory on Day 201 is shown in Figure 1 in relation to the reference stations.

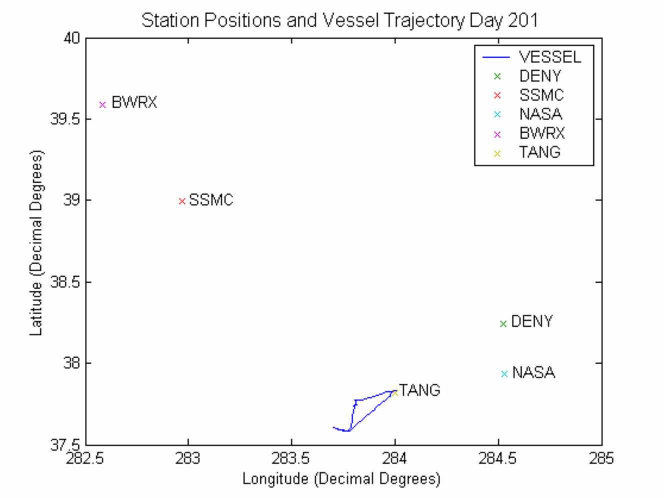


Figure 1 Vessel trajectory on Day 201 with respect to reference station locations

The distances between the reference stations and the vessel varied from approximately 1 to over 200 km.. The approximate distances from the reference stations to the vessel while at the dock are as follows: TANG 1 km., NASA 47 km., DENY 65 km., SSMC 157 km., and BWRX 197 km..

Data Processing and Analysis

The position solutions for the GPS receiver antenna mounted on board a NOS vessel were determined with respect to each reference station in post processing. All of the data presented were collected at 1 Hz. by a dual frequency GPS receiver. Broadcast orbits were used in all cases. The vessel solution using the primary reference station is the fixed integer "truth" solution. The vessel solutions using the remaining reference stations are ionospheric delay-free (non integer) solutions.

The inter-frequency carrier phase data combinations used in the GPS data processing were the narrow-lane, wide-lane and ionospheric delay-free linear combinations. Any inter-frequency linear combination can be represented by the general form:

$$N_L = nN_{L1} + mN_{L2} \quad (1)$$

where n and m are arbitrary numbers. By analogy, the ambiguity of this new observable N_L is:

$$N_L = nN_{L1} + mN_{L2} \quad (2)$$

The proper expression for the narrow-lane (N_{Ln}), the wide-lane (N_{Lw}) and the ionospheric delay-free (N_{Lc}) can be obtained by assigning appropriate values to n and m as follows.

N_{Ln}: n= m=1,
N_{Lw}: n= 1; m= -1
N_{Lc}: n= 1; m= $-f_{L1}/f_{L2}$

This results in:

$$N_{Ln} = N_{L1} + N_{L2} \quad (3)$$

$$N_{Lw} = N_{L1} - N_{L2} \quad (4)$$

$$N_{Lc} = N_{L1} - (f_{L2}/f_{L1})\,N_{L2} \quad (5)$$

Also, that N_{Ln} and N_{Lw} are integer numbers, whereas N_{Lc} is not. Their respective wavelengths are λ_{Lc}= 0.484 metres, λ_{Lw}= 0.862 metres, and λ_{Ln}= 0.107 metres. Wide-lane and narrow-lane have found use in ambiguity resolution. Several authors have shown their properties, whether used independently or not (Wubbena, 1989; Abidin et al., 1992; Han & Rizos, 1997). The ability to perform of ambiguity resolution decreases with increasing baseline length. Typically, a distance of 10 to 15 km. (between reference station and roving receiver) is considered as the limit for a successful narrow lane ambiguity resolution (Santos et al, 2000). Extending the range of ambiguity resolution has been the target of many investigations (e.g. Han, 1997; Kim & Langley, 2001).

The TANG to vessel solution is a narrow lane integer fixed solution. This integer fixed vessel solution is taken as truth and is used to compare with the vessel solutions from the other reference stations. This is reasonable since the narrow-lane solution is expected to be accurate to the centimetre level and such a small error will not compromise the conclusions of this study. All of the other stations, i.e. the four comparison reference stations (DENY, SSMC, BWRX, and NASA), were processed using an ionospheric delay-free carrier and code processing technique due to the length of their baselines. Each of the reference station to boat solutions will be compared against the fixed TANG to boat solution in order to determine the magnitude of the differences in latitude, longitude, and height. The epochs before solution convergence and beyond the narrow lane integer fixing capabilities of the filter were not included. The station coordinates of the comparison reference stations were each determined relative to TANG station ensuring relative consistency among the coordinates. As stated above these relative solutions are believed to be accurate to better than 5 cm.. Such an error would not compromise the results of this study.

The data from the remaining reference stations to the vessel were processed with a carrier triple difference-code double difference approach. From Wells et al (1986) and Hofmann-Wellenhof et al (1992), in relative GPS positioning several kinds of differences can be made between the carrier phase observation equations. A between receiver single difference is the difference between the phase equation for a receiver at one location and a satellite and the corresponding phase equation for a second receiver at another location and the same satellite. This effectively eliminates the satellite clock error and reduces the error associated with orbit and some of the atmospheric delay. A double difference is the subtraction of one single differenced phase equation from another at the same epoch. This effectively eliminates the receiver clock error. This double differencing allows for the elimination of both the satellite and receiver clock terms from the model equations. Triple differencing is the difference between epochs of double differences. This offers the advantage of removing the ambiguous integer number of phase cycles (N) from the triple differenced carrier phase equation. This means that the equation is not affected by changes in the number of phase cycles caused by cycle slips (Hofmann-Wellenhof et al, 1992; Remondi, 2000). This method may be less accurate than some integer fixing techniques but offers a very reliable solution for longer baselines (The XYZ's of GPS, 2001). The Kalman filter used to process the data using this technique is described by Remondi and Brown (2000).

It has been shown by Wells et al. (1987) that undifferenced carrier phase solutions and differenced ones are mathematically equivalent, provided that the errors and biases are properly modeled. Typically, the double difference solutions can take advantage of using the strength of fixing the ambiguity (provided that it is fixed to the correct value). This may be an advantage with respect to the triple difference. Nevertheless, the triple difference may offer a reliable solution for longer baselines because in this case ambiguity resolution (to its correct value) can be extremely difficult to obtain.

Results

A vessel solution was derived using each of the five reference stations. The narrow lane fixed integer solution produced using TANG station was taken as truth and the other solutions were subtracted from it by common epoch. This resulted in differences in latitude, longitude, and height for each of the other reference receivers on each of the three days. The mean value of the vessel solution differences, along with the deviation from (or about) the mean at 68% and 95% confidence levels, in each of latitude, longitude, and height can be seen in Tables 1, 2 and 3 for Days 201, 202, and 203 respectively.

Table 1 Values of mean and deviation from the mean at 68% and 95% confidence levels for differences Day 201 (in metres)

Base Station	Latitude			Longitude			Height		
	Mean	Deviation		Mean	Deviation		Mean	Deviation	
		68%	95%		68%	95%		68%	95%
NASA	-0.0078	0.0478	0.0937	0.0021	0.0607	0.1190	0.0076	0.1147	0.2248
DENY	0.0046	0.0366	0.0717	0.0017	0.0298	0.0584	0.0727	0.0582	0.1141
SSMC	0.0006	0.0489	0.0958	-0.0152	0.0556	0.1090	-0.0399	0.1289	0.2526
BWRX	-0.0034	0.0619	0.1213	-0.0157	0.0629	0.1233	0.0008	0.1623	0.3181

Table 2 Values of mean and deviation from the mean at 68% and 95% confidence levels for differences Day 202 (in metres)

Base Station	Latitude			Longitude			Height		
	Mean	Deviation		Mean	Deviation		Mean	Deviation	
		68%	95%		68%	95%		68%	95%
NASA	-0.0094	0.0535	0.1049	0.0065	0.0871	0.1707	-0.0846	0.1420	0.2783
DENY	0.0217	0.0389	0.0762	-0.0364	0.0633	0.1241	-0.0932	0.1135	0.2225
SSMC	0.0222	0.0377	0.0739	-0.0683	0.0582	0.1141	-0.2081	0.0832	0.1631
BWRX	0.0150	0.0360	0.0706	-0.0700	0.0564	0.1105	-0.1777	0.0762	0.1494

Table 3 Values of mean and deviation from the mean at 68% and 95% confidence levels for differences Day 203 (in metres)

Base Station	Latitude			Longitude			Height		
	Mean	Deviation		Mean	Deviation		Mean	Deviation	
		68%	95%		68%	95%		68%	95%
NASA	-0.0182	0.0632	0.1239	-0.0325	0.0316	0.0619	0.0467	0.1075	0.2107
DENY	0.0357	0.0668	0.1309	-0.0675	0.0512	0.1004	0.0529	0.0867	0.1699
SSMC	0.0306	0.0498	0.0976	-0.0883	0.0397	0.0778	-0.0546	0.0971	0.1903
BWRX	0.0166	0.0504	0.0988	-0.0761	0.0536	0.1051	-0.0232	0.1136	0.2227

The deviations from (or about) the mean of the differences, at 95% confidence level, for the latitude, longitude, and height components of the solutions for Days 201, 202, and 203 is presented graphically in Figure 2.

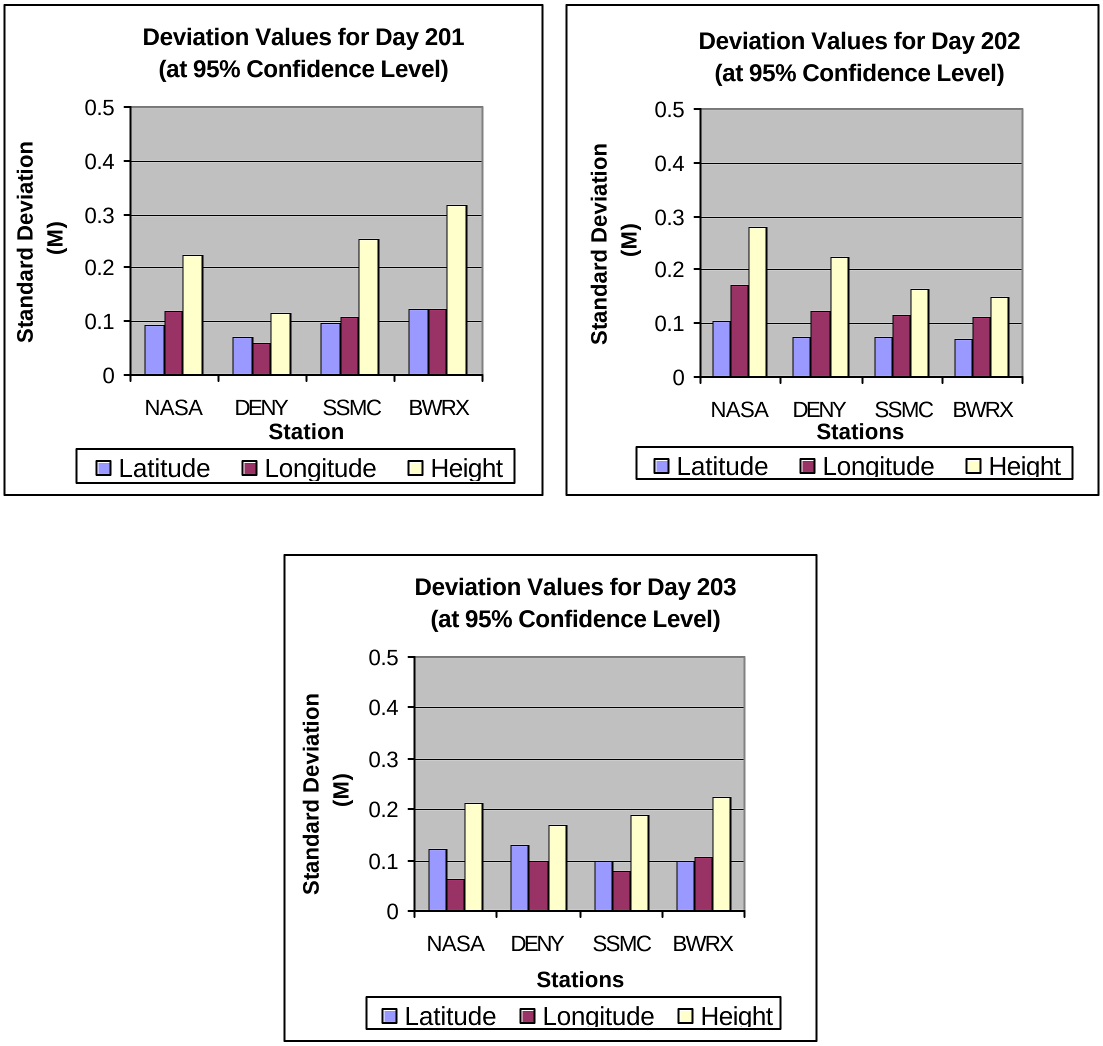
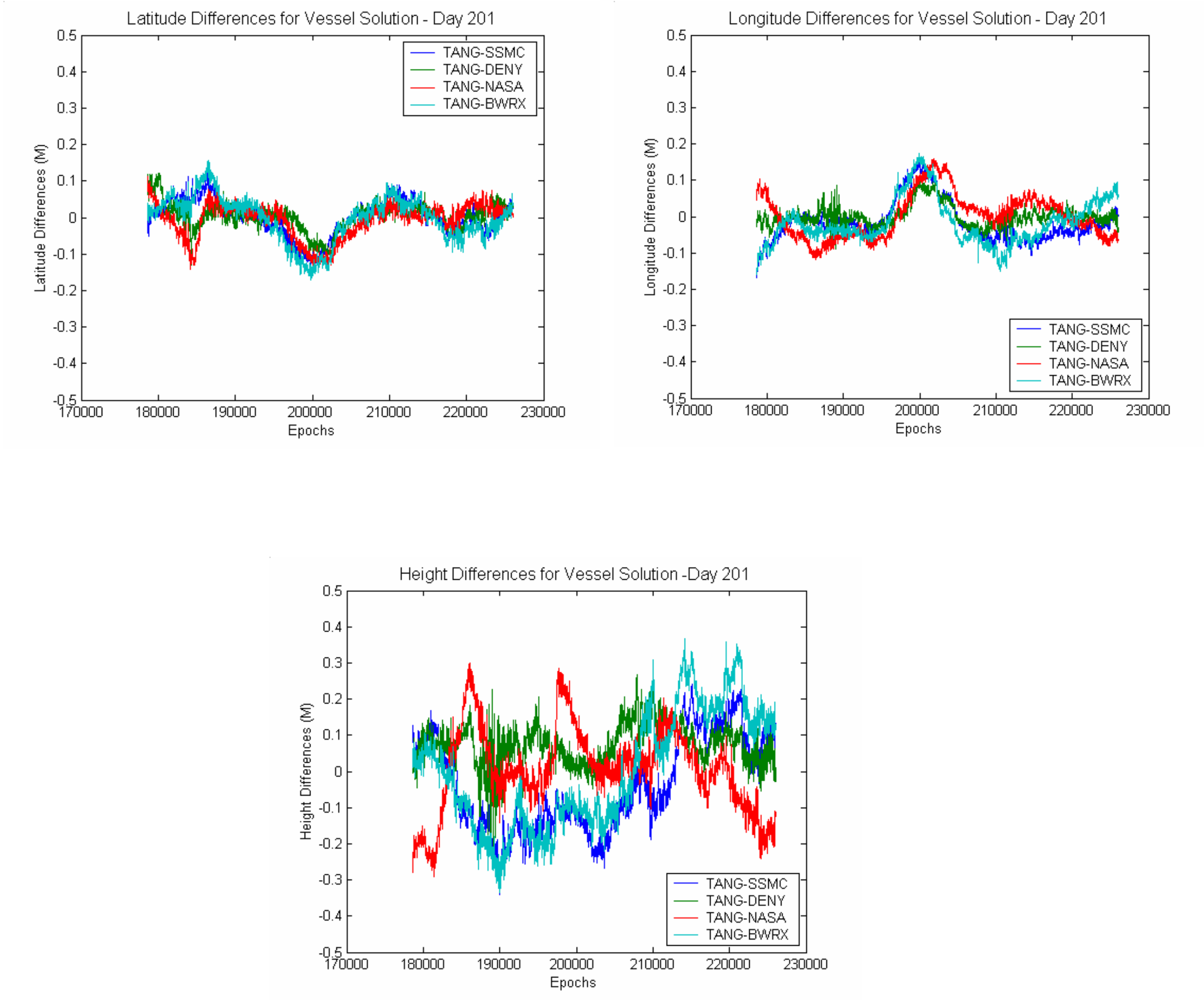


Figure 2 Values for deviation about the mean for differences in latitude, longitude and height for Days 201, 202, and 203 at 95% confidence level.

The distances from each station to the vessel while in dock are 47 km. for NASA, 65 km. for DENY, 157 km. for SSMC, and 197 km. for BWRX. The epochs correspond to GPS seconds of week.

The results of the processing were all consistent with expectations of reaching centimetre and decimetre agreement to the “truth” solution. Figures 3, 4, and 5 provide plots of the differences at one sigma in latitude, longitude, and height for Days 201, 202, and 203.



Figures 3 Differences in Latitude, Longitude and Height for each vessel solution from truth by epoch for Day 201.

On Day 201, latitude and longitude solutions seem to agree well both with the “truth” solution and with each other. Some areas of larger discrepancies are visible, for example between the epochs of 195000 and 205000 on Day 201. The height solutions are much more variable over the entire day due to the fact that GPS height geometry is much weaker than GPS horizontal geometry. In general, the horizontal component solutions agree within 10 cm. and the vertical component solutions agree within 20 cm. of the “truth”. The vessel begins moving away from the dock around epoch 223160 and travels to a distance of approximately 10 km. away from dock. There does not seem to be any noticeable negative effect on the solutions when the vessel is moving.

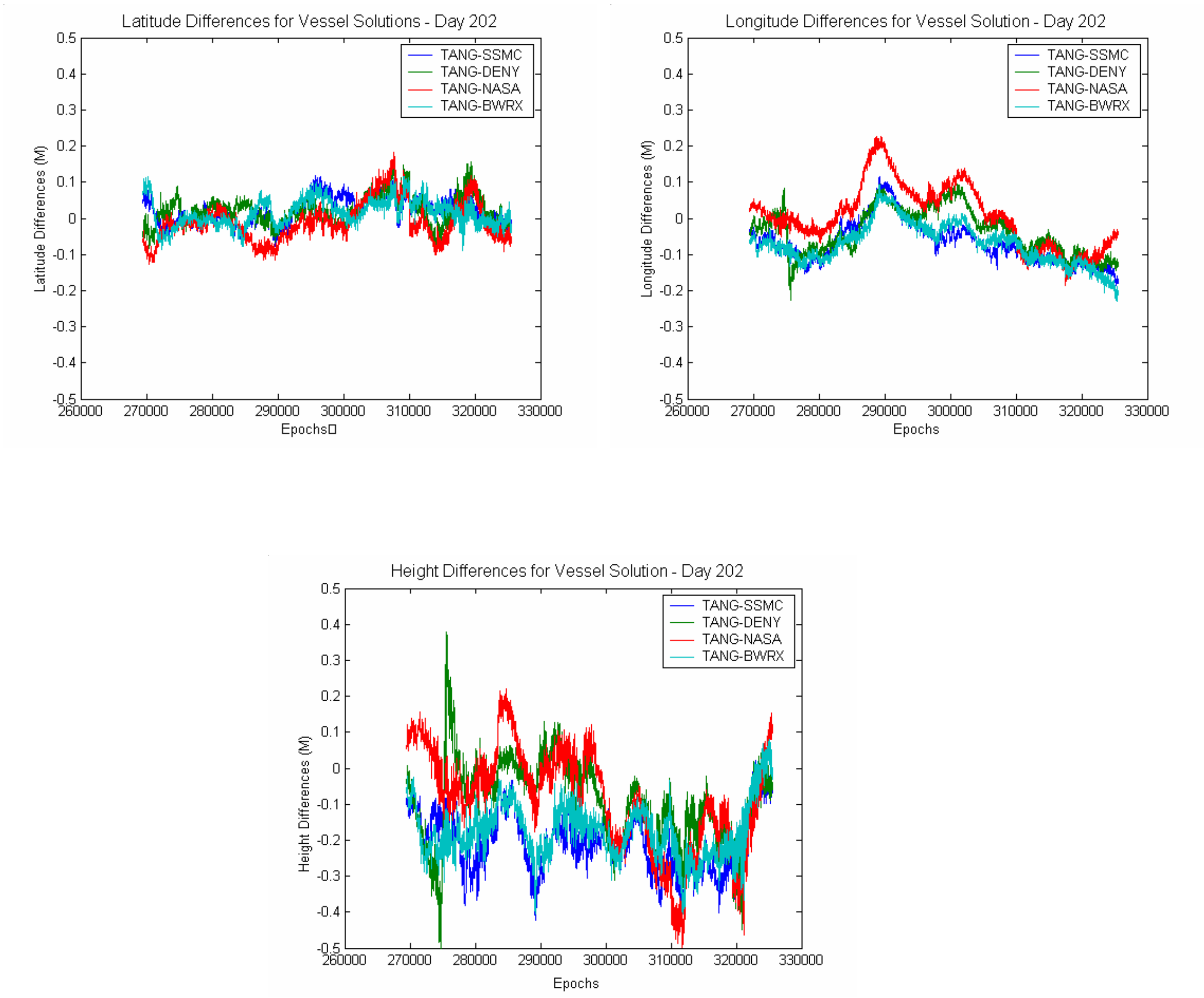


Figure 4 Differences in Latitude, Longitude and Height for each vessel solution from truth by epoch for Day 202.

On Day 202, there seems to be good agreement for the latitude solutions both with “truth” and with each other. The solutions, especially NASA, for the longitude seem to be biased from “truth”. There is also a pronounced bias in the height solutions. This bias can likely be attributed to the effect of troposphere due to poor weather conditions. There seems to be better agreement between the solutions for NASA and DENY, the stations that are closer to the vessel, and the solutions for SSMC and BWRX, the stations that are farther from the vessel. The horizontal solutions generally agree within 10 cm. and the vertical solutions within 20 cm.. The vessel begins moving away from the dock around epoch 307000 and travels to a distance of approximately 12 km. away from dock. The movement does seem to have a noticeable effect on the solution in the longitude; the solutions seem to start drifting as the vessel moves away from the dock.

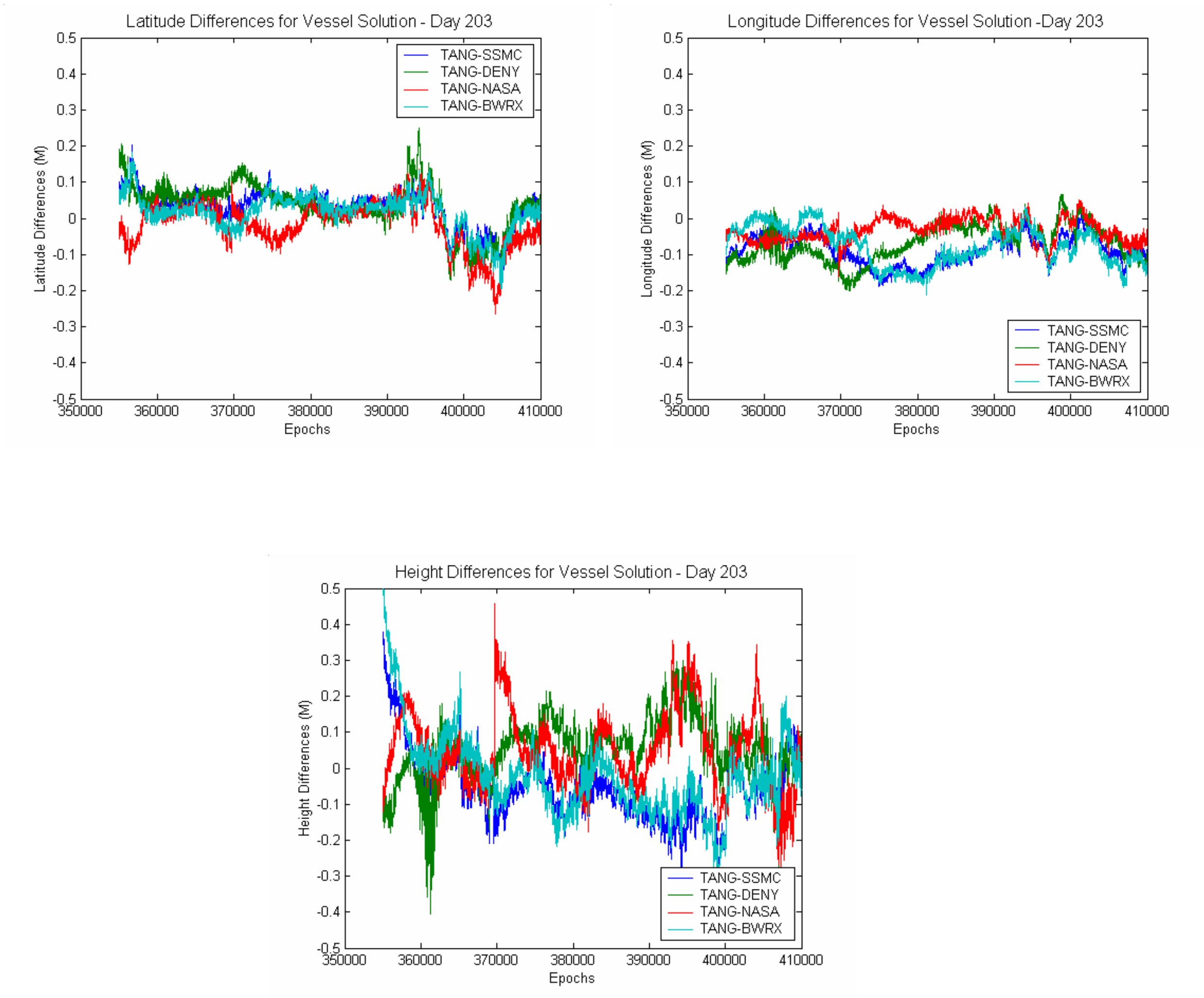


Figure 5 Differences in Latitude, Longitude and Height for each vessel solution from truth by epoch for Day 203.

There are two main areas of interest in the horizontal solutions for Day 203. The latitude solutions all seem to disagree with the "truth" in the time span after epoch 395000 to the end of the data set. There is a noticeable bias in the longitude solutions from "truth". There is also a time period, between 370000 and 390000, where the different longitude solutions seem to diverge. The horizontal solutions agree generally within 10 cm. and the vertical solutions agree generally within 20 cm.. The vessel begins moving away from the dock around epoch 405000 and travels to a distance of approximately 8 km. away from dock. There does not seem to be any negative effect on the solutions due to the vessel moving away from the dock.

In general, the solutions for the two closer stations, NASA and DENY, and the two further away stations, SSMC and BWRX, tended to agree more closely with each other. This could be related to having similar magnitudes of error for similar baseline distances or it could be related to the location of the stations. As can be seen in Figure 1, DENY and NASA are both located to the east and SSMC and BWRX are located to the north-west. A weather front could possibly affect one pair of stations but not the others.

Summary and Discussion

The differences in latitude, longitude, and height found for the vessel solutions from the "truth" solution were in the decimetre range. This is consistent with the accuracies for the ionospheric delay-free processing technique used. For the most part the magnitude of the errors varied based on baseline length but, other conditions, such as the weather, seemed to have an influence also. The data were collected under poor weather conditions with high humidity levels, high temperatures and during the first part of the period of high ionospheric activity. All of the solutions are also subject to a bias associated with orbit error. As broadcast orbits were used, the error would be expected to be on the order of low centimetres over several hundred kilometres (Wells et al, 1986).

The ionospheric delay-free solutions show results that are better than the achievable accuracies associated with using code DGPS. The results also emphasize the trade-off between the reliable and robust solution associated with an ionospheric delay-free solution versus the higher accuracies associated with an integer fixed solution. In situations where integer fixing is risky, due to baseline distance or other factors, ionospheric delay-free processing is a viable alternative for achieving the decimetre level accuracies required by many applications.

It should noted, once more, that this analysis and paper are an extension of the work presented in Huff and Remondi (2000). The data set used in this analysis has been and will continue to be a very valuable resource for analyzing GPS processing techniques and positioning results. Further analysis with this data is planned with the goal of incorporating detailed atmospheric data into the processing for improved tropospheric modeling.

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