SEQUENTIAL TIDAL HEIGHT PREDICTION USING ARTIFICIAL NEURAL NETWORK

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Abstract

Traditionally, tidal prediction was carried out using the harmonic method, which is based on the identification of the harmonic constituents existing in the tidal record. Unfortunately, however, unless long tidal records are available at the tide gauges, some important tidal constituents may not be identified. This, in turn, deteriorates the accuracy of the tidal prediction. Additionally, tidal prediction is affected by the accuracy of the estimated amplitudes and phases of tidal constituents.

To overcome the limitations of the traditional method, a neural network-based model is developed for sequentially predicting the tidal heights using tidal data series collected at various tide gauges. A modular three-layer feedforward neural network trained using the back-propagation algorithm is used for this purpose. Tide data from three tide gauges are used to validate the model. A comparison is made between the developed neural network model and the sequential least squares method for tidal prediction. It is shown that the accuracy level of the tidal prediction has improved by a factor of 5 when using the neural network model.

1. Introduction

Accurate tide information (current and predicted) is a key element for safe marine navigation in shallow waters as well as for other marine operations. Tidal prediction was conventionally carried out using various methods, with the harmonic method being the most common (Tianhang, 1991). With the harmonic method, the tide data series collected at a tide gauge is analysed to yield the amplitudes and the phase-lags of all the important harmonic tidal constituents with respect to those of the equilibrium tide. These tide constituents are then properly combined to predict the tidal heights at future times (Forrester, 1983). Depending on the number of constituent amplitudes and phase-lags taken in the prediction model, various accuracy levels can be expected. To be accurate, all the important constituents must be included in the prediction model, which requires long tidal records at the tide gauge, ideally spanning a period of more than 18.6 years. Unfortunately, long data series is not always available, which may lead to either large residuals or unstable solution, as interference between neighbouring constituents is expected (Forrester, 1983; Tianhang, 1991).

Tianhang (1991) developed a sequential least-squares prediction method, which allows for the modifications of the original solution of the tidal constituents due to the addition of new observations and parameters. Although reduces the computation time significantly compared to the batch method, this method exhibits large residuals practically when short tidal records are used.

To overcome the above limitations, a neural network-based model for sequential tidal prediction is developed in this paper. The motivation behind the selection of neural networks lies in their ability to model highly non-linear functions and mapping through supervised learning by example. A modular three-layer feedforward neural network trained using the back-propagation algorithm is used for this purpose. The design parameters of this model are the number of neural network inputs, outputs, and hidden nodes, as well as the structure of the feedforward network itself. Tide data from three tide gauges, namely Saint John, Yarmouth, and Charlottetown, are used to validate the model. A comparison is made between the developed neural network model and the sequential least-squares method. It is shown that the accuracy level of the tidal prediction has improved by a factor of 5 when using the neural network model.

2. Traditional Tidal Prediction Methods

As mentioned above, tidal prediction was traditionally carried out by the harmonic method, which is based on the identification of the harmonic tidal constituents existing in the tidal record. The tidal observation equation can be written as (Forrester, 1983; Tianhang, 1991):

$$\begin{aligned} h(t_i) &= Z_0 + \sum_{j=1}^{n} H_j cos(w_j t + q_j - f_j) \\ &= Z_0 + \sum_{j=1}^{n} P_j cos(w_j t + q_j) + Q_j sin(w_j t + q_j) \end{aligned} \tag{1}$$

where $h(t_i)$ is the observed tidal height, t_i is the Greenwich mean time, $t \in \{ t_1, t_2, \ldots, t_N)$, N is the number of tidal records, Z_0 is an unknown parameters representing the mean water level, n is the number of the harmonic constituents, H_j and f_j are the amplitude and phase-lag of the jth constituent, w_j and q_j are the equilibrium constituents and their Greenwich phases, and $P_j = H_j \, cos \, f_j$, and $Q_j = H_j \, sin \, f_j$. Equation (1) can be written in a more compact form for the entire tidal records as:

$$\boldsymbol{h} = \boldsymbol{A}\,\boldsymbol{x} \tag{2}$$

where $\boldsymbol{h} = [h(t_1), h(t_2), \ldots, h(t_N)]^T$ is the N×1 vector of the observed tidal heights, $\boldsymbol{x} = [Z_0, P_1, Q_1, \ldots, P_n, Q_n]^T$ is the (2n+1)×1 vector of the unknown parameters, and $\boldsymbol{A}$ is the design matrix given by:

$$\boldsymbol{A} = \begin{bmatrix} 1 & cos(w_1 t_1 + q_1) & \Lambda & sin(w_n t_1 + q_n) \\ 1 & cos(w_1 t_2 + q_1) & \Lambda & sin(w_n t_2 + q_n) \\ \mathrm{M} & \mathrm{M} & \Lambda & \mathrm{M} \\ 1 & cos(w_1 t_N + q_1) & \Lambda & sin(w_n t_N + q_n) \end{bmatrix} \tag{3}$$

The least-squares solution of the unknown parameters $\boldsymbol{x}$ can be obtained when N>2n+1 as follows:

$$\hat{\boldsymbol{x}} = (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{h} \tag{4}$$

The covariance matrix of the determined parameters $\hat{\boldsymbol{x}}$ can be estimated by applying the law of covariance propagation to (4). Once the least-squares solution for $\boldsymbol{x}$ is obtained, the predicted tidal heights $\hat{h}(t_i)$ can be obtained at any desired time t_i from (1). Tianhang (1991) modified the above method to allow for the tidal prediction to be determined in a sequential manner. His algorithm accommodates the addition of new observations as well as new tidal constituents (see Tianhang, 1991 for more details). To test his model, he predicted the tidal values over one day using approximately one-week of tidal records at three stations, namely Saint John, Yarmouth, and Halifax. The maximum residual error (the difference between predicted and true values) was approximately one meter, which, as would be expected, becomes larger with the increases in time.

3. Artificial Neural Network

Artificial neural networks (ANN) are computational models that imitate the human brain in performing a particular task. They are capable of solving complex problems through learning, or training, and then generalizing the network outputs for other inputs. A neural network consists of processing elements, or neurons p_i, that are massively interconnected. Each of the connecting links is characterized by its own weight, w_{ki} (Figure 1). An activation function, e.g., a sigmoid function, is applied to limit the amplitude of the neuron, while an external bias, b_k, is applied to increase or lower the net input of the activation function (Haykin, 1999). The network is trained to find the optimal values for the weights and biases.

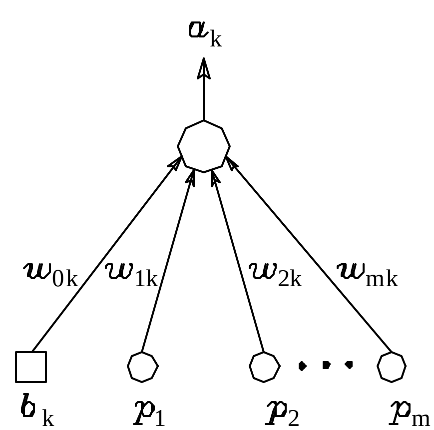

Figure 1. A simple neuron model.

Artificial neural networks can be designed in various ways, depending on how the neurons are structured and the learning algorithms, or rules, used. Network architectures may be classified as single-layer feedforward, multi-layer feedforward, and recurrent networks (Haykin, 1999). Initially, we selected the traditional non-modular feedforward structure for our prediction model. Unfortunately, however, this structure failed to learn the required mapping. Consequently, a modular three-layer feedforward neural network trained using the back-propagation algorithm was selected (Figure 2). The back-propagation algorithm uses the gradient descent in weight space to minimize the output error. It converges to a locally optimal solution when an adequate number of input-output pairs of samples are subjected to the network for an adequate number of epochs. The back-propagation algorithm can be summarized in three steps as follows (see Haykin, 1999 for more details). The first step is to propagate the input forward through the network. The neurons in the first layer receive external inputs:

$$\boldsymbol{a}^0 = \boldsymbol{p} \tag{5}$$

$$\boldsymbol{a}^{m+1} = \boldsymbol{f}^{m+1}(\boldsymbol{W}^{m+1}\boldsymbol{a}^m + \boldsymbol{b}^{m+1}), \quad \text{for } m = 0, 1, \ldots, M\text{-}1 \tag{6}$$

Where M is the number of layers in the network. Equation (5) provides the starting point for equation (6), and the output of the layer m is the input to the layer $m+1$. The outputs of the neurons in the last layer are considered as the network outputs:

$$\boldsymbol{a} = \boldsymbol{a}^M \tag{7}$$

The next step is to calculate the output error and *back-propagate* it to calculate the sensitivities (***s*** values).

$$\boldsymbol{s}^M = -2\dot{\boldsymbol{F}}^M(\boldsymbol{n}^M)\,(\boldsymbol{t} - \boldsymbol{a}) \tag{8}$$

$$\boldsymbol{s}^m = \dot{\boldsymbol{F}}^m(\boldsymbol{n}^m)\,(\boldsymbol{W}^{m+1})\,\boldsymbol{s}^{m+1}, \quad \text{for } m = M\text{-}1, \ldots, 2, 1 \tag{9}$$

Finally, the weights and biases are updated, based on the calculated sensitivities, using the approximate steepest descent rule:

$$\boldsymbol{W}^m(k+1) = \boldsymbol{W}^m(k) - \alpha\boldsymbol{s}^m(\boldsymbol{a}^{m-1})^T \tag{10}$$

$$\boldsymbol{b}^m(k+1) = \boldsymbol{b}^m(k) - \alpha\boldsymbol{s}^m \tag{11}$$

Where α is the step size. The three steps are repeated until the error reaches a minimum; hence, the calculated weights of the connections would represent the solution network.

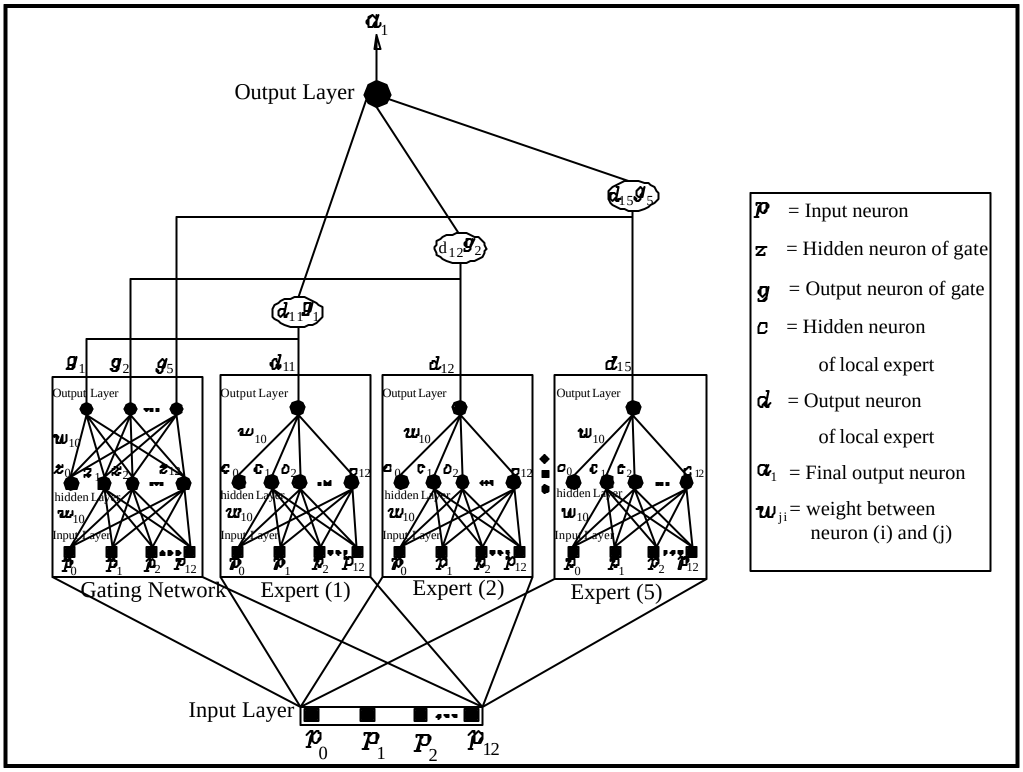

Figure 2. A modular neural network with the structure [12-25-5-1].

4. Results and discussions

Hourly Tidal data from three tide gauges, namely Saint John, Yarmouth, and Charlottetown, were used to validate the neural network model. The structure of the neural network was built using the NeuralWare software (NeuralWare, 2001). Several tests were conducted to optimize the structure of the neural network for each of the three data series. It was concluded that the modular neural network with the structure [12-25-5-1] gives the best results, i.e., has the lowest root-mean-square (RMS) error. Initially, we used the variable time t as the only input to the network, which is usually followed in the literature (see, for example, El-Rabbany et al., 2002). However, although the trained network represented the training dataset reasonably well, the network prediction was rather poor. Therefore, we followed another approach, which was suggested by Schuh et al. (2002). In this approach, the immediate past values of tidal records are used as input to the network, while future values of tidal records are used as the desired output. In the subsequent epochs, the training patterns are time-shifted as shown in Figure 3.

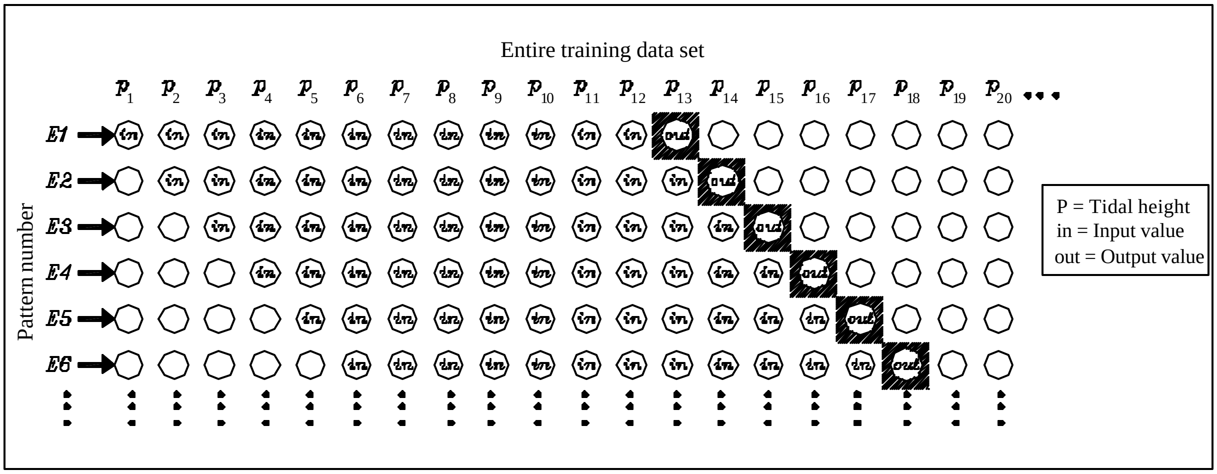

Figure 3. Training patterns used in the neural network input.

Each of the data series was divided into three datasets: training testing and validation datasets (Figure 4). The first 1000 hourly tidal records were assigned to the testing dataset, while the last 1000 hourly tidal records were assigned to the validation dataset. The training dataset was selected to represent the middle portion of the data series, which varied from one tide gauge to another (see Figure 4 and Table 1). The training was stopped based on the testing the generalization performance the neural network using the testing dataset. Table 1 shows the number of iterations needed to train the network for each of the data series. After training and testing the network, we generalized the model to predict ahead the last 1000 values of the data series and compared the results with the observed tidal records. This was done in a sequential manner to emulate the real time condition. Figure 5 and 6 show the predicted tidal heights versus the desired (i.e., actual) values, and the prediction errors (i.e., residuals) for Yarmouth. Similar results were obtained for the other two stations (see Table 1). It can be seen that the maximum prediction error is approximately 20 cm, with the bulk of the residuals fall within the +/-10 cm range. This shows that the developed neural network model has the capability to precisely predict the tidal values, even when long tidal records are not available.

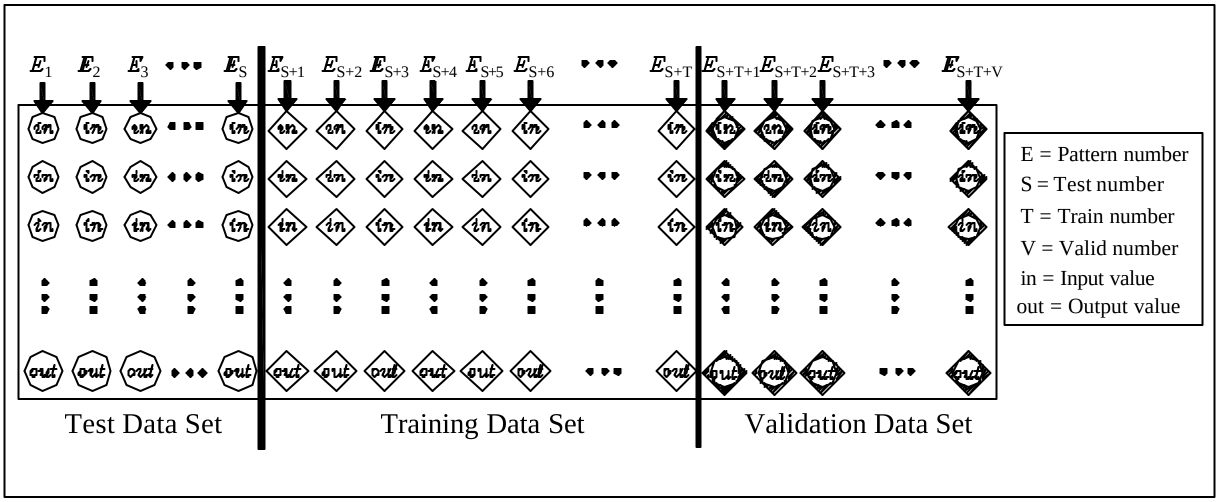

Figure 4. Selection of testing, training, and validation datasets.

Table 1. Results of long time series record (more than one year)

Stations / Sympols	Saint John	Yarmouth	Charlottetown
Data span From: dd/mm/yyyy To : dd/mm/yyyy	01 / 08 / 2000 31 / 12 / 2001	01 / 03 / 1996 31 / 12 / 1997	01 / 06 / 1997 31 / 10 / 1999
N (Total set)	12432	16104	21192
N_S (Testing set)	1000	1000	1000
N_T (Training set)	10432	14104	19192
N_V (Validation set)	1000	1000	1000
Strucure	[12-25-5-1]	[12-25-5-1]	[12-25-5-1]
RMS (Testing)	0.012720	0.020085	0.018811
RMS (Training)	0.012145	0.016763	0.021292
RMS (Validation)	0.011533	0.021207	0.024277
CORR (Testing)	0.999503	0.998475	0.997872
CORR (Training)	0.999375	0.998774	0.997186
CORR (Validation)	0.999557	0.998169	0.996499

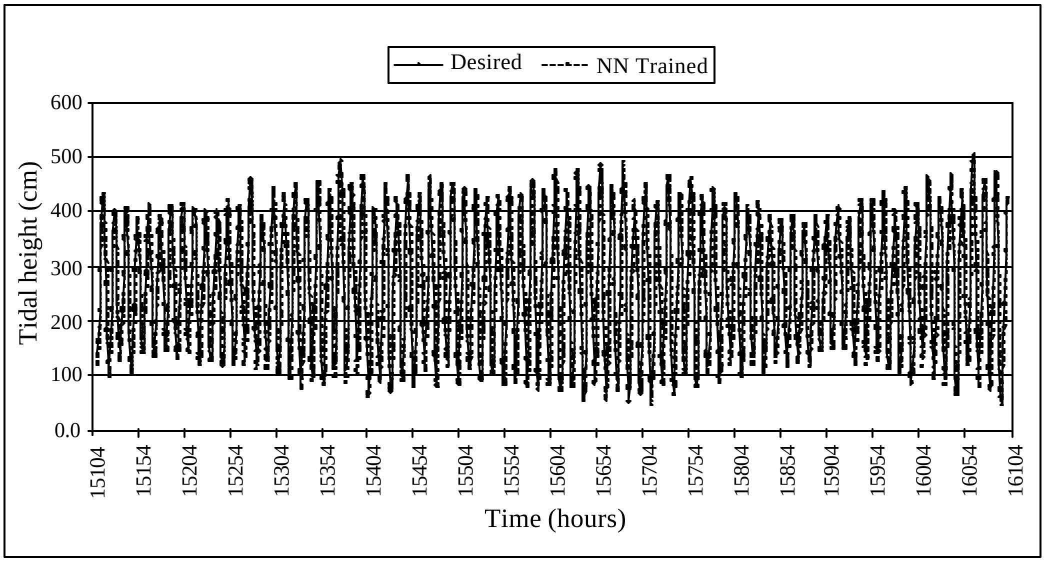

Figure 5. Predicted tidal heights versus desired (actual) values for Yarmouth.

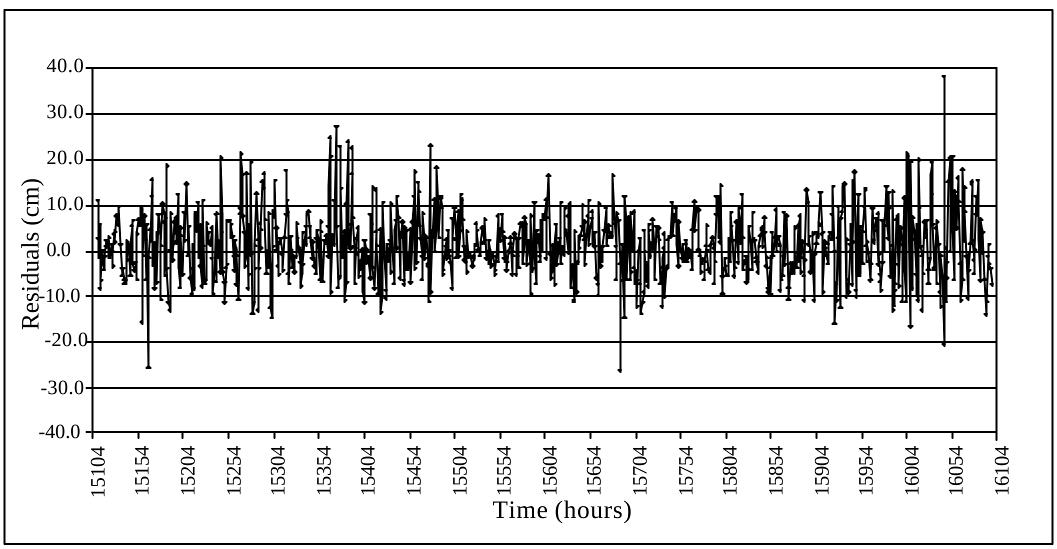

Figure 6. Prediction errors for Yarmouth.

To further validate our model, we compared it with the sequential least squares tidal prediction model developed by Tianhang (1991). To do this, we trained the network using a randomly selected one-week of tidal records at the three tide gauges mentioned above. Our goal was to sequentially predict the tidal values on the eighth day, i.e., the day which follows the one-week training. Figures 7 and 8 show the predicted tidal heights versus the desired (i.e., actual) values, and the prediction errors (i.e., residuals) for Yarmouth. Similar results were obtained for the other two stations (see Table 2). It can be seen that, despite the very short data span, the maximum prediction error is only 20 cm, which is five times lower than the sequential least squares tidal prediction model.

Table 2. Results of short time series record (one-week)

Sympols \ Stations	Saint John	Yarmouth	Charlottetown
N (Total set)	216	216	216
N_S (Testing set)	24	24	24
N_T (Training set)	168	168	168
N_V (Validation set)	24	24	24
Strucure	[12-25-5-1]	[12-25-5-1]	[12-25-5-1]
RMS (Testing)	0.013195	0.014893	0.037837
RMS (Training)	0.009910	0.018571	0.020091
RMS (Validation)	0.020693	0.020090	0.033559
CORR (Testing)	0.999467	0.999565	0.994441
CORR (Training)	0.999753	0.999264	0.998929
CORR (Validation)	0.999654	0.998870	0.996733
Residual (Max.)	+ 20.652 cm	+ 17.591 cm	+ 10.232 cm
Residual (Min.)	- 20.802 cm	- 16.899 cm	- 12.878 cm

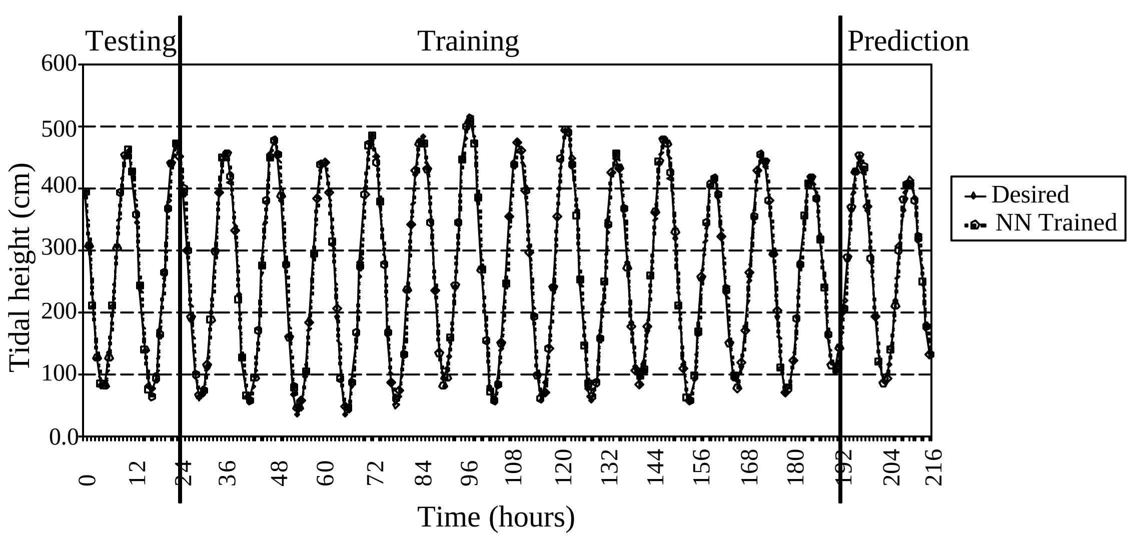

Figure 7. Predicted tidal heights versus desired (actual) values for Yarmouth [based on one-week tidal observations].

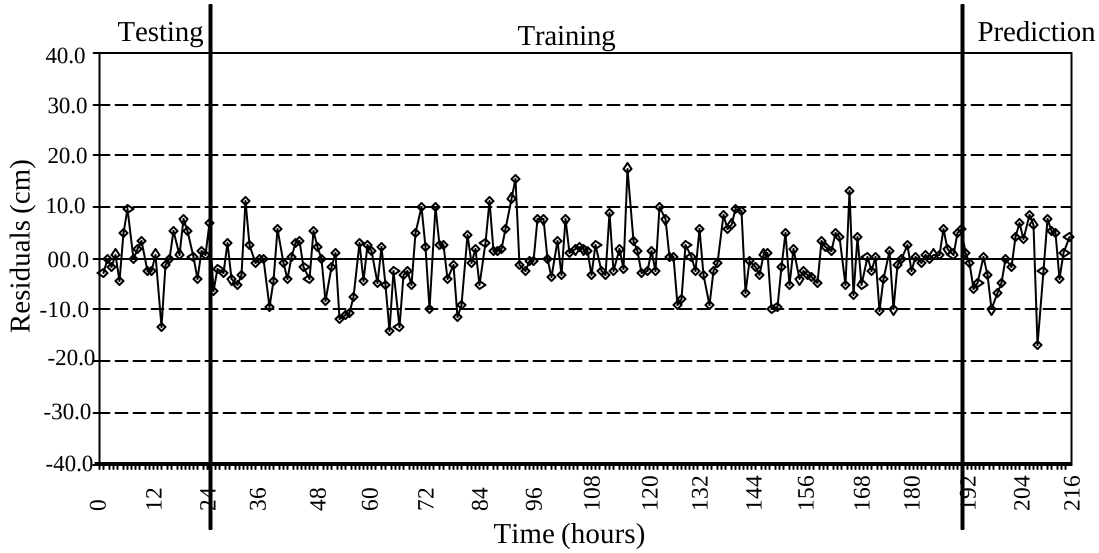

Figure 8. Prediction errors for Yarmouth [based on one-week tidal observations].

Conclusions

A sequential tidal prediction model using Artificial Neural Networks was developed in this paper. The modular neural network structure gave the best performance results (i.e., the minimum RMS), and therefore was used in predicting the tidal values. Hourly tide data, with varying lengths, from three tide gauges were used to verify the model. It is shown that the maximum prediction error is approximately 20 cm, with the bulk of the residuals fall within the +/-10 cm range. A performance comparison was made between the developed neural network model and the sequential least squares method for tidal prediction. Three randomly selected data series with one-week duration each were used for that purpose. It is shown that, despite the very short data span, the maximum prediction error is still 20 cm with the neural network model, which is five times lower than the sequential least squares model. This shows that the developed neural network model has the capability to precisely predict the tidal values in a sequential manner, even when long tidal records are not available.

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